Hale School

Mathematics Specialist

Term 2 2017

Test 4 - Integration

SECTION ONE

/ 30

Name: _____

Instructions:

- **SECTION ONE: CAS calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of SECTION ONE: 30 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 1 **(9 marks)**

Determine the following integrals:

a) $\displaystyle\int \frac{\sin^2(2x) + \cos^2(2x)}{\cos^2(4x)}\,dx$ (3 marks)

b) $\displaystyle\int \frac{\cos^3 x}{3}\,dx$ (3 marks)

c) $\displaystyle\int_0^1 \frac{e^{ax}}{1+e^{ax}}\,dx$ where a is a constant. (3 marks)

Question 2 **(8 marks)**

Using the substitution $x = 2\cos u$ determine the following definite integral:

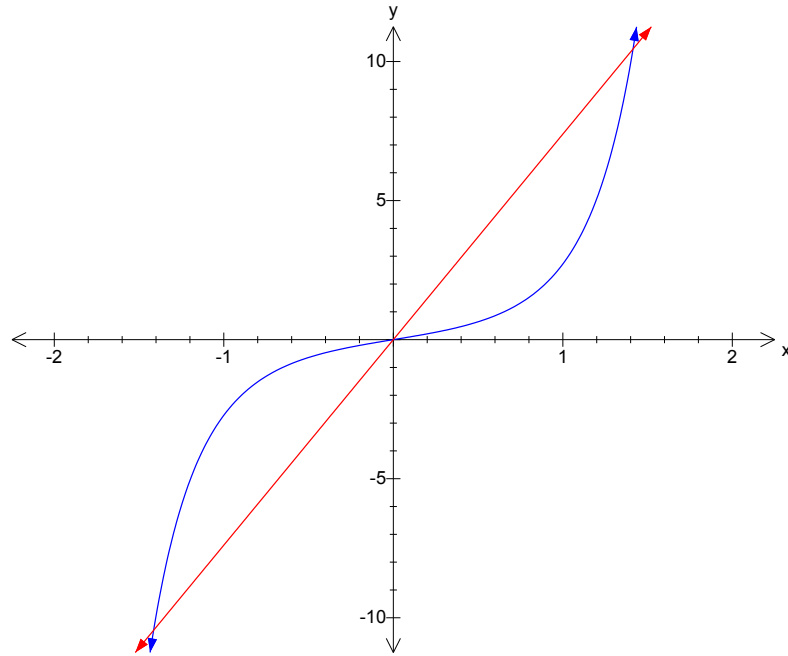$$\int_0^1 \sqrt{4 - x^2} \; dx$$

Question 3 (8 marks)

Determine the following integral:

$$\int \frac{3x^2 + 13x - 16}{3x^2 + 2x - 8} \, dx$$

Question 4 (5 marks)

The graphs defined by $y = xe^{x^2}$ *and* $y = e^2 x$ are shown below. Calculate the **exact** area enclosed between the two curves.

5

Hale School

Mathematics Specialist

Term 2 2017

Test 4 - Integration

SECTION TWO

/13

Name: _____

Instructions:

- **SECTION TWO: CAS calculators are allowed**
- **External notes are not allowed**
- **Duration of SECTION TWO: 15 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 5 **(7 marks)**

A sphere of radius 10 cm is formed by rotating the semi-circle $y = \sqrt{100 - x^2}$ about the *x*-axis.

a) Using volume of revolution determine the volume of this sphere.
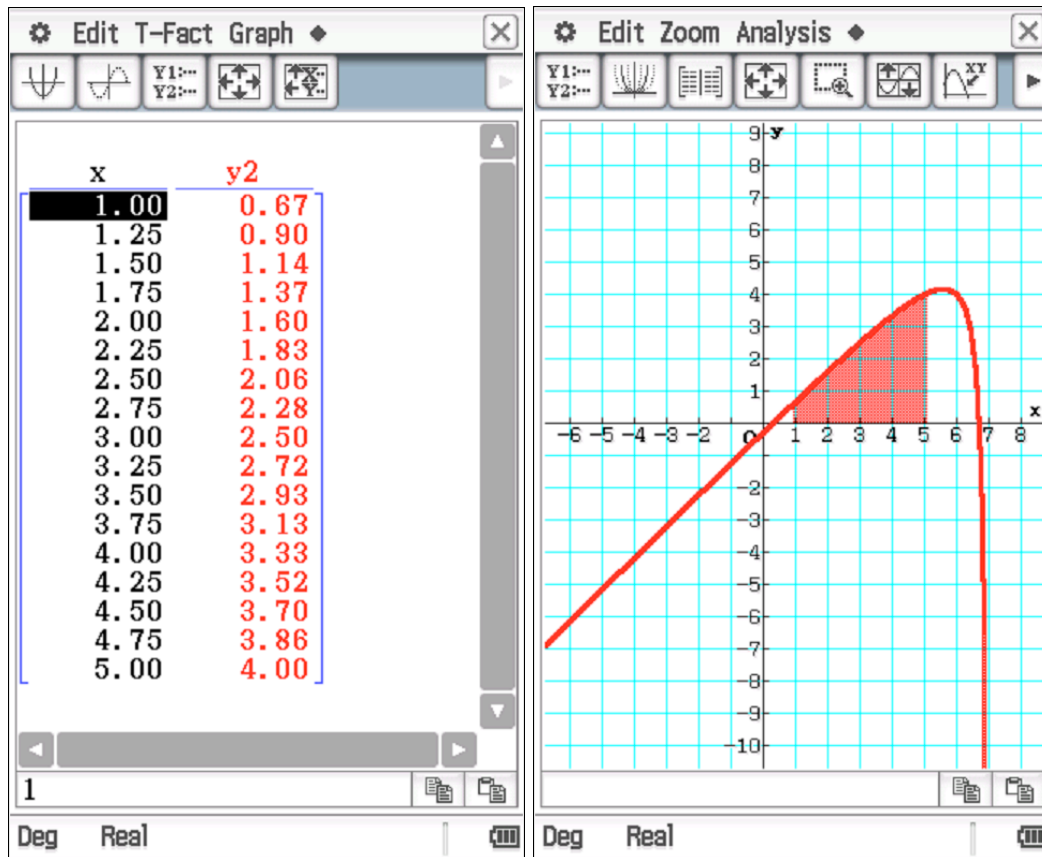
(2 marks)

b) The sphere of radius 10 cm has a cap of height 2 cm removed from the top. Find the volume of the spherical cap. (2 marks)

c) The remaining portion of the sphere has a cylindrical hole of radius 1 cm bored symmetrically from the top of the cut sphere, directly through the centre to the other side. Find the volume of the sphere remaining. (3 marks)

Question 6 (6 marks)

The shaded region R in the diagram above shows the region trapped between the curve $y=f(x)$, the x-axis and the lines $x=1$ and $x=5$. The accompanying table shows the value of the function $f(x)$ (indicated as y2 from the calculator input) for the various values of x.

a) Estimate the area of R using the trapezium rule with 8 strips. (3 marks)

b) Estimate the area of R using Simpson's rule with 4 strips. (3 marks)

END OF TEST